PRESSURE DEPENDENCE OF THE THERMAL CONTACT RESISTANCE FOR ROUGH SURFACES

V. S. Novikov

UDC 536.241.001

The contact resistance is expressed analytically as a function of load for elastic and elasto-plastic contacts by taking the rough surface as consisting of spherical projections with a normal distribution.

Compression is one method of adjusting thermal contact resistance [1,2]. Here we derive an analytic expression for the load dependence of the resistance via the geometrical and mechanical features of rough surfaces.

It is always possible [3, 4] to replace contact between two rough surfaces by contact without allowance with a rough one whose projections have the variance

$$\sigma^2 = \sigma_1^2 + \sigma_2^2, \tag{1}$$

in which σ_i^2 represents the variance for the individual surfaces.

The following equation has been given [1] for the thermal resistance of a contact without allowance for heat transfer by radiation and via the intervening medium:

$$R_c = \frac{1}{\pi a n \lambda} \operatorname{arctg} \frac{r-a}{a} \equiv \frac{1}{n} R_n, \tag{2}$$

which has been derived on the assumption of a uniform distribution of equal circular contacts. Load increase raises a and n but reduces r, with the sizes of the individual circles having a certain spread around the mean a, so that the simple summation used in deriving (2) should be replaced by integration on the basis of the statistical features.

Elastic Contact. It has been shown [3,4] that a normal distribution represents closely the distribution of the heights of the roughness relative to a standard plane, while a single projection may be represented as a very shallow spherical one. Then [3] the load P_i acting on a ridge produces a deformation

$$b_i = 0.83 \left[\frac{P_i (1-\mu^2)}{E\sqrt{R}} \right]^{2/3}. \tag{3}$$

The area of an individual contact is [4] related to b_i by

$$f_i = \pi R b_i, \quad f_i = 2\pi R b_i \tag{4}$$

for elastic and plastic contacts respectively. In the absence of a load, height of the largest ridge equals the distance between the standard plane and a smooth plane. A load brings the surfaces together, and this distance becomes d, so all ridges with heights exceeding d will be in contact with the smooth surface, and the number of contacts is

$$n = \frac{N}{\sigma\sqrt{2\pi}} \int_d^{z^*} \exp\left(-\frac{z^2}{2\sigma^2}\right) dz. \tag{5}$$

Institute of Technical Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 19, No. 1, pp. 62-67, July, 1970. Original article submitted July 18, 1969.

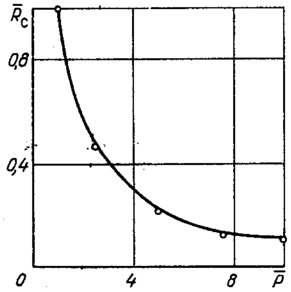

Fig. 1. Relative thermal resistance of contact R_e/R_c^* versus relative load P/P^* ($P^* = 2.026 \cdot 10^6$ N/m^2; $R_c^* = R_c|_{P = P^*}$, m$^2 \cdot$deg/W). Points show experimental data [1].

The properties of a normal distribution allow one to replace the upper limit of integration by infinity ($z^* \to \infty$), so

$$n = \frac{MN}{\sqrt{2\pi}} \frac{\sigma}{d} \exp\left(-\frac{d^2}{2\sigma^2}\right),$$
$$M \equiv 1 - \frac{1}{2\left(\frac{d}{\sqrt{2}\sigma}\right)^2} + \frac{1 \cdot 3}{4\left(\frac{d}{\sqrt{2}\sigma}\right)^4} - \cdots \tag{6}$$

Rough values have been given [4] for d/σ as a function of load. In general, the contacts are unevenly distributed over unit area of the nominal surface, but the very simple assumption of a uniform distribution implies that the following is the radius of the region from which heat flow lines converge on a contact:

$$r = \frac{1}{2}\sqrt{\frac{1}{n}}. \tag{7}$$

The conductance of unit area of nominal surface is the sum of the conductances of the individual contacts, i.e.,

$$\alpha_1 = \frac{N}{\sigma\sqrt{2\pi}} \int_d^{z^*} R_n^{-1} \exp\left(-\frac{z^2}{2\sigma^2}\right) dz. \tag{8}$$

The actual area of contact in elastic contact is a few per cent of the nominal, so we expand $\arctan[(r-a)/a]$ as a series to get

$$\alpha_1 = 2\sqrt{\frac{2}{\pi}} \lambda N\sigma\sqrt{R} \exp\left(-\frac{d^2}{2\sigma^2}\right) \times \left(\frac{1}{3}\frac{M}{d}\sqrt{z^* - d} + \frac{1-M}{\pi r}\sqrt{R}\right), \tag{9}$$

where M is defined by (6) and r by (7). The distance d between the surfaces in (9) can be expressed via the external load as follows for the above model

$$P = \left(\frac{1}{0{,}83}\right)^{3/2} \frac{NE\sqrt{R}}{\sqrt{2\pi}\sigma(1-\mu^2)} \int_d^{z^*} (z-d)^{3/2} \exp\left(-\frac{z^2}{2\sigma^2}\right) dz, \tag{10}$$

which is the integral of P_i over all the load-bearing projections. Integration gives

$$P = \frac{2}{3\sqrt{2\pi}} \left(\frac{1}{0.83}\right)^{3/2} \frac{NE\sigma(1-M)}{1-\mu^2} \sqrt{R(z^* - d)} \exp\left(-\frac{d^2}{2\sigma^2}\right). \tag{11}$$

It is best to deduce d(P) graphically because (11) is transcendental.

Similarly, for α_1, P, and r we can derive more convenient equivalent expressions if we calculate the integral of (5) as a Poisson integral, with

$$\alpha_1 = 2\sqrt{\frac{2}{\pi}} \lambda N\sqrt{R}\left[\sqrt{\frac{\pi}{2}}\left(\frac{1}{3}\sqrt{z^* - d} - \frac{d\sqrt{R}}{\pi r}\right)\right.$$

$$\times \operatorname{erfc} x + \frac{\sigma \sqrt{R}}{\pi r} \exp(-x^2) \Big], \tag{12}$$

$$P = \frac{1}{3}\sqrt{\frac{2}{\pi}}\left(\frac{1}{0.83}\right)^{3/2} \frac{NE\sigma}{1-\mu^2}$$

$$\times \sqrt{R(z^*-d)}\left[\exp(-x^2) - \sqrt{\pi}\, x \operatorname{erfc} x\right],$$

$$r = (2N \operatorname{erfc} x)^{-1/2}, \quad \operatorname{erfc} x = 1 - \Phi(x), \quad x \equiv \frac{d}{\sqrt{2}\sigma},$$

where $\Phi(x)$ is the probability integral, tabulated in [5].

We have an exponential dependence on d for $R_c = 1/\alpha_1$, which is confirmed by experiment, as in the P dependence in (11) [1,2]. For d small (P large) we have

$$d \approx \sigma\sqrt{2}\left\{\ln\left[\frac{1}{3}\sqrt{\frac{2}{\pi}}\left(\frac{1}{0.83}\right)^{3/2} \frac{NE\sigma\sqrt{Rz^*}}{P(1-\mu^2)}\right]\right\}^{1/2}, \tag{13}$$

and α_1 is linearly dependent on P:

$$\alpha_1 = 2\sqrt{\frac{2}{\pi}}\, \lambda N \sqrt{R}$$

$$\times\left[\frac{1}{3}\sqrt{\frac{1}{2}\pi z^*} + 3\sqrt{\frac{1}{\pi}}\, 0.83^{3/2} \frac{(1-\mu^2)P}{E\sqrt{Nz^*}}\right]. \tag{14}$$

This also agrees with experiment. It follows from (9) and (14) that the contact resistance increases with the roughness.

Elastoplastic Contact. At high levels the projections attain a critical strain b* where plastic strain starts [3]. The following is the pressure at the center of the contact circle between an elastic sphere and a rigid plane:

$$q = 0.918\left[\frac{P_i E^2}{4R^2(1-\mu^2)}\right]^{1/3}. \tag{15}$$

The strain becomes plastic if q exceeds the critical value $q^* = c\sigma_s$, where c is 1-6 and is a factor incorporating the projection shape and the interaction of projections. Then when

$$d^* = z^* - b^* = z^* - \frac{0.83\sqrt[3]{16}}{0.918^2} Rc^2\sigma_s^2\left(\frac{1-\mu^2}{E}\right)^2, \tag{16}$$

we get plastic deformation of the highest projections.

If the nominal pressure is less than the yield point, the plastically deformed contacts bear [4] a ratio $\exp(-b^*/\sigma)$ to the total number, so (4) gives the conductance of unit area of nominal contact as

$$\alpha_2 \approx \alpha_1\left[1 + \exp\left(-\frac{b^*}{\sigma}\right)\right]. \tag{17}$$

Also, σ_s falls as the temperature in the contact zone increases, so the critical strain also decreases, and the conductance increases because there are more plastically deformed projections.

To compare the calculated relation with experimental data we used the measurements given in [1] (p. 135) for contact between pairs of 1Kh18N9T steel surfaces: $\lambda = 17.7$ W/m·deg, $H_{ro} \approx \sigma = 11.7 \cdot 10^{-4}$ cm, $E = 1.9 \cdot 10^{11}$ N/m^2 (at 200°C), and $\mu = 0.3$. Unfortunately, the other necessary parameters are not given in [1]. We estimated the values $N = 1.5 \cdot 10^5$ cm^{-2}, $R = 50 \cdot 10^{-4}$ cm, $z^* = 44 \cdot 10^{-4}$ cm and $b^* = 0.4 \cdot 10^{-4}$ cm. Various methods of surface treatment tend to give identical roughness height, but with different dispositions and deformation resistances for the projections and, as the surface treatment was not stated in [1], the uncertainty in N, R, and z* should be eliminated by comparing theoretical and experimental relationships in relative units. The unit employed for $\bar{P}$ in Fig. 1 was $P^* = 2.026 \cdot 10^6$ N/m^2, while that for $\bar{R}_c$ was the thermal resistance at $P = P^*$. Figure 1 shows that the calculated $R_c(P)$ agrees with experiment, though the R_c deduced from (17) with the above parameters were somewhat higher than those of [1].

Boundary Conditions in Contact Heat Transfer. These results can be used as boundary conditions where contact resistance is important, e.g. friction, machining, or the theory of thermal stress. They are readily extended to electrical contacts and heat transfer by radiation and via the intervening media. Let the last two factors be characterized by the heat-transfer coefficients α_r and α_f; then the overall coefficient of contact conductance is

$$\alpha = \alpha_j + (1 - \varepsilon_j)(\alpha_f + \alpha_r), \quad j = 1, 2. \tag{18}$$

For an elastic contact

$$F_1 = \sqrt{\frac{\pi}{2}}\, RN\left[\sigma \exp(-x^2) - \sqrt{\frac{\pi}{2}}\, d \operatorname{erfc} x\right], \tag{19}$$

and for an elastoplastic one

$$F_2 = F_1\left[1 + \exp\left(-\frac{b^*}{\sigma}\right)\right]. \tag{20}$$

The definition of N makes F_j equal to ε_j.

This method can easily be applied to other models for the surface.

NOTATION

σ	standard deviation of projection heights;
R_c	contact thermal resistance;
a, r	radii of contact spot and of contraction region;
N, n	total number of projections and cavities and number of contact spots per unit area of nominal contact surface;
λ	effective thermal conductivity of contact zone;
z, z*	height and maximum height of projections;
P_i, b_i	load per projection and deformation;
E, μ, σ_s	elastic modulus, Poisson's ratio and yield limit for projection material;
f_i	area of a contact point;
F_j, ε_j	relative, actual contact area;
d	distance between smooth surface and standard plane of rough surface;
α_1, α_2	contact conductivity of elastic and elastoplastic contacts;
R	calculated radius of projection;
P	load on contacting surfaces;
b*	critical deformation of projection;
q, q*	contact pressure at the centre of projection and value at critical deformation;
α_f, α_r	coefficients of heat transfer by intermediate fluid and radiation.

LITERATURE CITED

1. Yu. P. Shlykov and E. A. Ganin, Contact Heat Transfer [in Russian], Gosenergoizdat, Moscow –Leningrad (1963).
2. W. B. Kouwenhouvent and J. H. Potter, Welding J., 27, No. 10, 515 (1948).
3. I. V. Kragel'skii, Friction and Wear [in Russian], Mashinostroenie, Moscow (1968).
4. J. Greenwood, Theoretical Principles of Engineering Calculations [Russian translation], No. 3, Mir, Moscow (1967).
5. B. I. Segal and K. A. Semendyaev, Five-Figure Mathematical Tables [in Russian], Fizmatgiz, Moscow (1959).